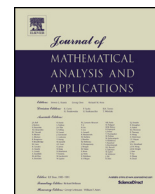




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Payne type inequalities for  $L^p$ -norms of the warping functions<sup>☆</sup>

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## ABSTRACT

Let  $u(x, G)$  be a warping function of a multiply connected plane domain  $G$ . A new physical functional of  $u(x, G)$  with an isoperimetric monotonicity property is constructed. It is proved that  $L^p$ - and  $L^q$ -norms of the warping function satisfy sharp isoperimetric inequalities, which, besides the norms, can contain the functional  $\mathbf{u}(G) = \sup_{x \in G} u(x, G)$ . As a particular case of one of these inequalities it follows the classical result of Payne for the torsional rigidity of  $G$ . Our proofs are based on the technique of estimates on level lines devised by L.E. Payne.

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## 1. Introduction

Let  $G$  be a multiply connected plane domain. We denote by  $\Gamma_0$  the outer boundary curve of  $G$ , and by  $\Gamma_1, \dots, \Gamma_n$  the internal boundary curves. The boundary-value problem that defines the warping function  $u(x, G)$  of  $G$  is

$$\begin{cases} \Delta u = -2 & \text{in } G, \\ u = 0 & \text{on } \Gamma_0, \\ u = c_i & \text{on } \Gamma_i, \ i = 1, \dots, n, \end{cases} \quad (1)$$

where the constants  $c_i$  are determined by the conditions

$$\oint_{\Gamma_i} \frac{\partial u}{\partial n} ds = -2a_i, \quad i = 1, \dots, n,$$

$\partial/\partial n$  is the inward normal derivative, and  $a_i$  is the area enclosed by  $\Gamma_i$ .

A classical result of the theory of partial differential equations [9] is that the boundary-value problem (1) has a positive and unique solution.

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